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GDAŃSK UNIVERSITY OF TECHNOLOGY

Jakub Golik, MSc

Game Theory in Economics and Everyday Life

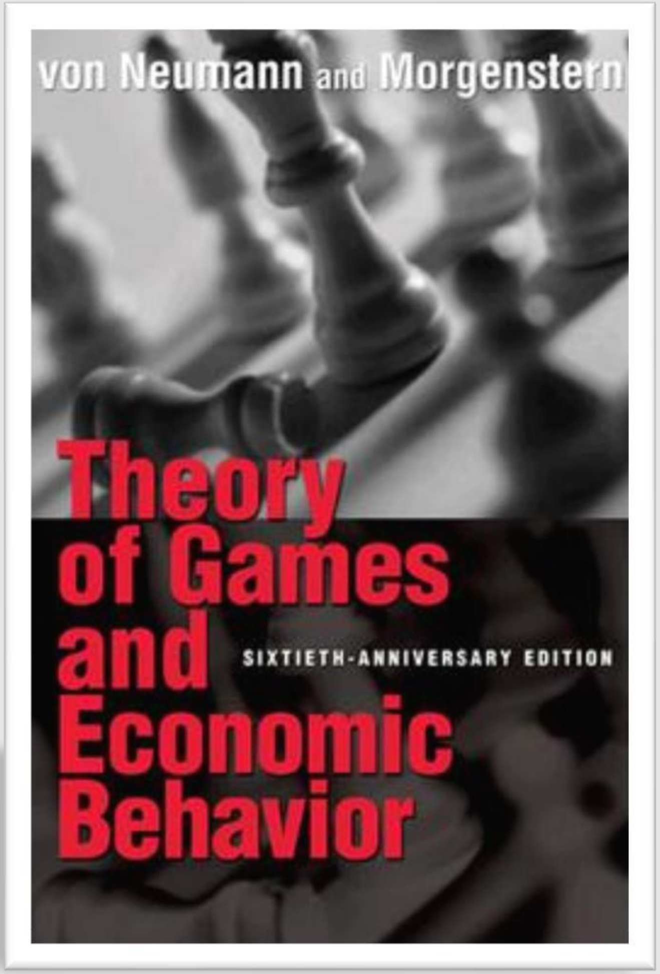
Jakub Golik

- **Research and Teaching Assistant**
  Department of Entrepreneurship
  - **Bachelor** in **Applied Mathematics** | Faculty of Technical Physics and Applied Mathematics
  - **Master** in **International Management** | Faculty of Management and Economics
  - **PhD student** in **Economics** | Faculty of Management and Economics
- Scientific interests:
  - Concept of **utility and risk propensity**
  - **Expected Utility Hypothesis / Prospect Theory**
  - **Decision Theory**
  - **Game Theory** in Economics

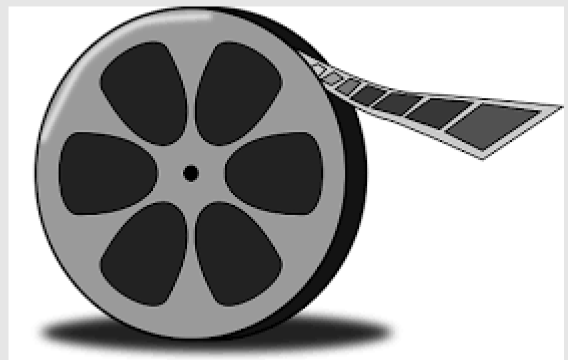
https://www.researchgate.net/profile/Jakub_Golik/

Game Theory

- formally a branch of Mathematics
- John von Neumann,
  Oskar Morgenstern
  Theory of Games and Economic Behavior,
  Princeton University Press (1944)
- applied in i.a. *Economics*, *Psychology*, *Political Sciences*, *Biology* (*Genetics*), *Computer Sciences* or *Philosophy*

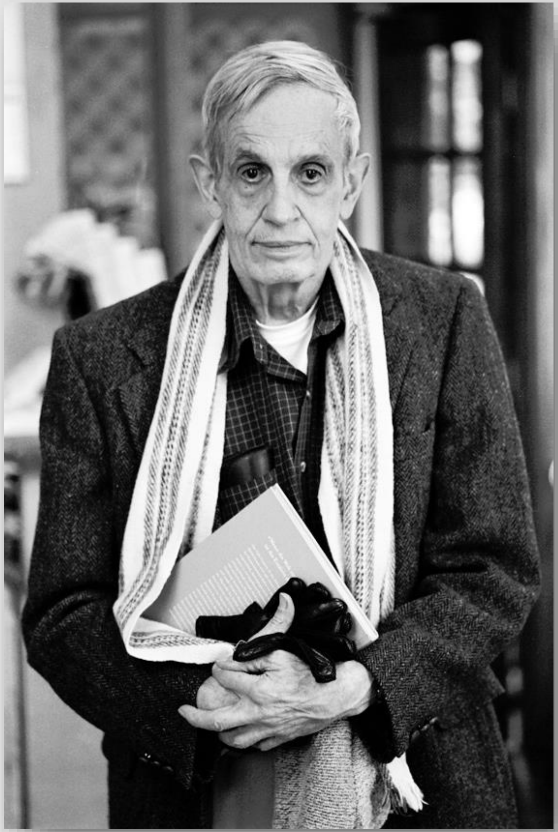


A Beautiful Mind (2001) – USA



John Forbes Nash Jr.

- 1928 – 2015 (age 86)
- Selected awards:
  - John von Neumann Theory Prize (1978)
  - **Nobel** Prize in **Economic Sciences** (1994)
  - Abel Prize (2015)
- known for ***Nash equilibrium***



Types of games

2-player	Simultanous	Normal Form	Constant sum Zero sum	Complete Information	Perfect Information	Symmetric	Non-Cooperative
n-player	Sequential	Extensive Form	Non-Zero Sum	Incomplete Information	Imperfect information	Asymmetric	Cooperative

Types of games

- ✓ Information asymmetry
- ✓ Perfect recall

2-player	Simultanous	Normal Form	Constant sum Zero sum	Complete Information	Perfect Information	Symmetric	Non-Cooperative
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Types of games

2-player	Simultanous	Normal Form	Constant sum Zero sum	Complete Information	Perfect Information	Symmetric	Non-Cooperative
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Perfect information vs **Complete** information

Perfect information (*usually sequential games*) - each player, when making any decision, is perfectly informed of all the events that have previously occurred, including the "initialization event" of the game (e.g. the starting hands of each player in a card game).

Complete information is importantly different from perfect information. In a game of complete information, the structure of the game and the payoff functions of the players are commonly known but players may not see all of the moves made by other players (for instance, the initial placement of ships in **Battleship**); there may also be a chance element (as in **most card games**).

<u>A game with complete information may or may not have perfect information, and vice versa!</u>

Chess

2-player	Simultanous	Normal Form	Constant sum Zero sum	Complete Information	Perfect Information	Symmetric	Non-Cooperative
n-player	Sequential	Extensive Form	Non-Zero Sum	Incomplete Information	Imperfect information	Asymmetric	Cooperative

Chess

2-player	Simultanous	Normal Form	Constant sum Zero sum	Complete Information	Perfect Information	Symmetric	Non-Cooperative
n-player	Sequential	Extensive Form	Non-Zero Sum	Incomplete Information	Imperfect information	Asymmetric	Cooperative

Name a game!

2-player	Simultanous	Normal Form	Constant sum Zero sum	Complete Information	Perfect Information	Symmetric	Non-Cooperative
n-player	Sequential	Extensive Form	Non-Zero Sum	Incomplete Information	Imperfect information	Asymmetric	Cooperative

Prisoner's Dilemma

Two members of a criminal gang are arrested and imprisoned. Each prisoner is in solitary confinement with no means of communicating with the other. The prosecutors lack sufficient evidence to convict the pair on the principal charge, but they have enough to convict both on a lesser charge. Simultaneously, the prosecutors offer each prisoner a bargain. Each prisoner is given the opportunity either to betray the other by testifying that the other committed the crime, or to cooperate with the other by remaining silent. The possible outcomes are:

- If A and B each betray the other, each of them serves two years in prison
- If A betrays B but B remains silent, A will be set free and B will serve three years in prison (and vice versa)
- If A and B both remain silent, both of them will serve only one year in prison (on the lesser charge).

Prisoner's Dilemma payoff matrix

B A	B stays silent	B betrays
A stays silent	-1 -1	0 -3
A betrays	-3 0	-2 -2

Prisoner's Dilemma

2-player	Simultanous	Normal Form	Constant sum Zero sum	Complete Information	Perfect Information	Symmetric	Non-Cooperative
n-player	Sequential	Extensive Form	Non-Zero Sum	Incomplete Information	Imperfect information	Asymmetric	Cooperative

LET'S PLAY Prisoner's Dilemma!

B / A	B stays silent	B betrays
A stays silent	A: -1, B: -1	A: -3, B: 0
A betrays	A: 0, B: -3	A: -2, B: -2

Nash equilibrium

- **Informal** definition
  - It is a "state of the game" such as no player can profitably deviate, given the actions of the other players.
  - If each player has chosen a strategy, and **no player can benefit by changing strategies** while the other players keep theirs unchanged, then the current set of strategy choices and their corresponding payoffs constitutes a **Nash equilibrium**.
- **Nash's Existence Theorem**
  - Every finite game has a mixed strategy Nash equilibrium.

Pareto Optimality

- Named after Vilfredo Pareto (Italian economist), Pareto optimality is a measure of efficiency. An outcome of a game is **Pareto optimal** if **there is no other outcome that makes every player at least as well off and at least one player strictly better off**.
- That is, a Pareto Optimal outcome cannot be improved upon without hurting at least one player. Often, a Nash Equilibrium is not Pareto Optimal implying that the players' payoffs can all be increased.

Prisoner's Dilemma Nash equilibrium

B / A	B stays silent	B betrays
A stays silent	-1 (B), -1 (A)	0 (B), -3 (A)
A betrays	-3 (B), 0 (A)	-2 (B), -2 (A)

Prisoner's Dilemma Nash equilibrium

A \ B	B stays silent	B betrays
A stays silent	A: -1, B: -1	A: -3, B: 0
A betrays	A: 0, B: -3	A: -2, B: -2

Prisoner’s Dilemma Nash equilibrium

B A	B stays silent	B betrays
A stays silent	-1 -1	0 -3
A betrays	-3 0	-2 -2

Is it Pareto Optimal?

Prisoner's Dilemma Nash equilibrium

A \ B	B stays silent	B betrays
A stays silent	A: -1, B: -1	A: -3, B: 0
A betrays	A: 0, B: -3	A: -2, B: -2

Is it Pareto Optimal?

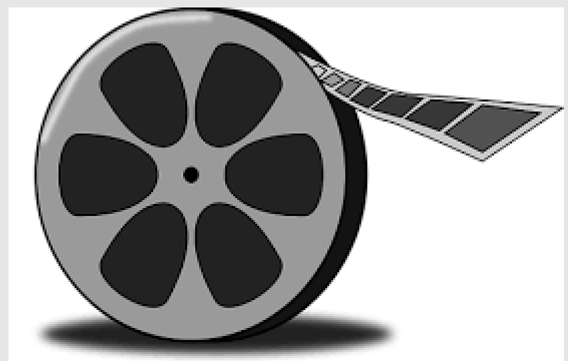
Prisoner’s Dilemma Generalization

A \ B	B stays silent	B betrays
A stays silent	R / R	S / T
A betrays	T / S	P / P

T > R > P > S

T – temptation payoff
R – reward for cooperation
P – punishment payoff
S – "sucker’s" payoff

The Dark Knight (2008) – USA /UK



Joker's Ferry Game ver. 1

B A	B Cooperates	B Detonates
A Cooperates	0 0	1 0
A Detonates	0 1	0 0

Joker's Ferry Game ver. 1

B / A	B Cooperates	B Detonates
A Cooperates	A: 0, B: 0	A: 0, B: 1
A Detonates	A: 1, B: 0	A: 0, B: 0

Joker's Ferry Game ver. 1

B / A	B Cooperates	B Detonates
A Cooperates	0 **0**	1 **0**
A **Detonates**	0 **1**	0 **0**

Joker's Ferry Game ver. 1

A \ B	B Cooperates	B Detonates
A Cooperates	A: 0, B: 0	A: 0, B: 1
A Detonates	A: 1, B: 0	A: 0, B: 0

Joker's Ferry Game ver. 1

B / A	B Cooperates	B **Detonates**
A Cooperates	0, **0**	0, **1**
A Detonates	1, **0**	0, **0**

Joker’s Ferry Game ver. 1 Nash equilibrium

B / A	B Cooperates	B **Detonates**
A Cooperates	0 / 0	1 / 0
A **Detonates**	0 / 1	0 / 0

Joker's Ferry Game 2 **Survival > Morality**

A \ B	B Cooperates	B Detonates
A Cooperates	1 \ 1	1 \ 2
A Detonates	2 \ 1	0 \ 0

Joker’s Ferry Game ver. 2 Nash equilibrium

B A	B Cooperates	B Detonates
A Cooperates	1 1	2 1
A Detonates	1 2	0 0

?

?

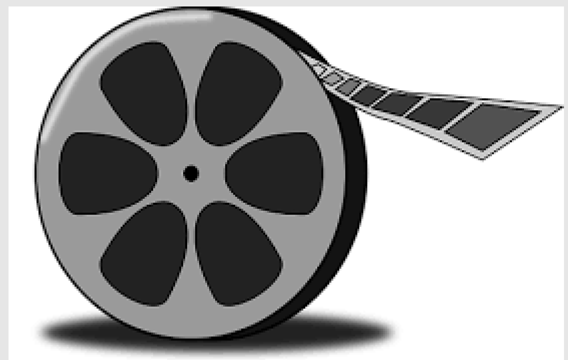
Joker’s Ferry Game 3 **Survival < Morality**

B A	B Cooperates	B Detonates
A Cooperates	2 2	1 2
A Detonates	2 1	0 0

Joker's Ferry Game ver. 3 Nash equilibrium

B A	B Cooperates	B Detonates
A Cooperates	2 2	1 2
A Detonates	2 1	0 0

The Dark Knight (2008) – PROLOGUE



The Pirate Game

There are 5 rational pirates (in strict order of seniority A senior to B, B to C, C > D and D > E) who found 500 gold coins.
They must decide how to distribute them.

The most senior pirate first proposes a plan of distribution. The pirates, including the proposer, then vote on whether to accept this distribution. If the majority accepts the plan, the coins are dispersed and the game ends. ***In case of a tie vote, the proposer has the casting vote****. If the majority rejects the plan, the proposer is thrown overboard from the pirate ship and dies, and the next most senior pirate makes a new proposal to begin the system again. The process repeats until a plan is accepted or if there is one pirate left.*

The Pirate Game

Pirates base their decisions on four factors:

1) Each pirate wants to **survive**.
2) Given survival, each pirate wants to **maximize the number of gold coins** each receives.
3) Each pirate would prefer to **throw another overboard**, if all other results would otherwise be equal.
4) The **pirates do not trust each other**, and will neither make nor honor any promises between pirates apart from a proposed distribution plan that gives a whole number of gold coins to each pirate.

Solution by **backward induction**

5 pirates: **A > B > C > D > E**
500 gold coins

Let’s think ahead and reason backwards!

- What would happen if the game continued so only **pirate E** remained?

Solution by **backward induction**

5 pirates: **A > B > C > D > E**
500 gold coins

Let's think ahead and reason backwards!

- What would happen if the game got to **pirate D** proposing a split?

Solution by **backward induction**

5 pirates: **A > B > C > D > E**
500 gold coins

Let’s think ahead and reason backwards!

- What would happen if **pirate C** is offering the split?

Solution by **backward induction**

5 pirates: **A > B > C > D > E**
500 gold coins

Let's think ahead and reason backwards!

- What would happen if **pirate C** is offering the split?
- Pirate C needs to buy 1 vote to make the plan go through. If pirate C dies, then pirate D would take all 500 coins and pirate E ends up with nothing. **<u>All the pirates know this</u>**. This presents an opportunity to buy the vote of pirate E.

Solution by **backward induction**

5 pirates: **A > B > C > D > E**
500 gold coins

Pirates C and E would vote for this plan and it would go through.

C	499
D	0
E	1

Solution by **backward induction**

5 pirates: **A > B > C > D > E**
500 gold coins

What would happen if **pirate B** is offering the split?

Pirates B and D vote affirmatively against C and E, and pirate B holds the tie-breaking vote so the plan goes through.

B	499
C	0
D	1
E	0

Solution by **backward induction**

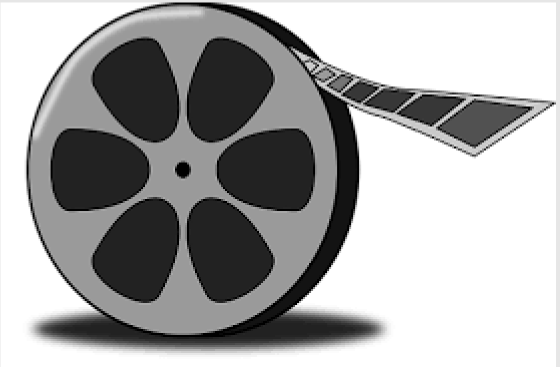
5 pirates: **A > B > C > D > E**
500 gold coins

Now we return to the original situation. What does **pirate A** do?

Pirates A, C, and E vote for the plan and it passes.

A	498
B	0
C	1
D	0
E	1

The Dark Knight (2008) – Mob negotiations *Ultimatum Game*



Recommended reading

- Nash Jr, J. F. (1950). The bargaining problem. *Econometrica: Journal of the Econometric Society*, 155-162.
- Schuster, S. (2017). A new solution concept for the ultimatum game leading to the golden ratio. *Scientific reports*, *7*(1), 5642.
- Einav, L. (2010). Not all rivals look alike: Estimating an equilibrium model of the release date timing game. *Economic Inquiry*, *48*(2), 369-390.
- Osborne, M. J., & Rubinstein, A. (1994). *A course in game theory*. MIT press. http://ebour.com.ar/pdfs/A%20Course%20in%20Game%20Theory.pdf
- works on Inductive Game Theory by prof. *Jeff Kline*

Sources / References

- Some ideas and some representations and solutions to described games were based on the following blog posts:

  - http://quantitativepeace.com/blog/2008/07/the-dark-knight.html
  - https://mindyourdecisions.com/blog/2016/07/12/the-pirate-game-game-theory-tuesdays/
  - https://mindyourdecisions.com/blog/2008/08/19/game-theory-in-the-dark-knight-a-critical-review-of-the-opening-scene-spoilers/

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Patronat Honorowy
Prezydent
Miasta Gdańska

Jakub Golik, MSc

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